
Discrete and Gradient Elasticity Cosserat Modeling of Granular Chains

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Abstract

The present study theoretically investigates the free vibration problem of a discrete granular system. This micro structured system consists of uniform grains elastically connected by shear and rotation springs. Such a granular structural system is confined by discrete elastic interactions, to take into account the lateral granular contributions. This discrete repetitive system could be considered as a discrete Cosserat chain or a lattice elastic model with shear interaction. The natural frequencies of this discrete system are exactly calculated for the granular beam resting on two supports, starting from the resolution of the linear difference eigenvalue problem. The natural frequencies of such a granular model are analytically calculated whatever the considered modes. It is shown that the discrete equations of this granular system, for an infinite number of grains, converge to the differential equations of the Bresse-Timoshenko beam resting on Winkler foundation (such a Bresse-Timoshenko beam can be also classified as a continuous Cosserat beam model). A gradient Bresse-Timoshenko model is constructed from continualization of the difference equations of the granular system. This continuous gradient elasticity Cosserat model is obtained from a polynomial or a rational expansion of the pseudodifferential operators, stemming from the continualization process. The natural frequencies of the continuous gradient Cosserat models are compared with those of the discrete Cosserat model associated with the granular chain. Scale effects of the granular chain are clearly captured by the continuous gradient elasticity model.

Keywords: Granular media, Discrete Cosserat, Continuum Cosserat, Vibration analysis

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